This paper contains a tabulation of the function

$$Z(x, y) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty}{}' \frac{e^{2\pi i(kx+ly)}}{k^2 + l^2}$$

where the prime denotes the fact that the term with $k = l = 0$ is omitted.

The table is given for values x and y in the range

$$\tfrac{1}{2} \geqq x \geqq y \geqq 0,$$

where x and $y = 0(0.01)0.5$. The entries are given to six and sometimes seven decimal places, and are said to be accurate to at least two units in the last decimal place.

In the calculation of this table, use was made of the seven-place tables of the exponential integrals published in 1954 by the U.S.S.R. Academy of Science, Institute of Exact Mechanics and Computation.

A. H. T.

9[L].—HERMAN H. LOWELL, *Tables of the Bessel-Kelvin Functions Ber, Bei, Ker, Kei, and their Derivatives for the Argument Range 0(0.01)107.50*, Technical Report R-32, National Aeronautics and Space Administration, Washington, D. C., 1959, 300 p., 26 cm.

These tables provide an elaborate and attractively arranged compilation of decimal values of the Bessel-Kelvin functions (frequently referred to simply as the Kelvin functions) of the first and second kinds of order zero, together with their first derivatives. Approximations to ber and bei and their first derivatives appear in floating-point form to generally 13 or 14 significant figures. On the other hand, the number of significant figures given for ker and kei and their first derivatives vary from 9 to 13, according to a pattern explained in the detailed introduction, which also describes the construction of these tables and the checks applied to the tabular entries. The calculations were performed on an IBM 650 calculator using the Bell Telephone Laboratories Double-Precision (16-figure) Interpretive System.

In addition to the checks applied by the author, the reviewer collated the values of ber x and bei x with similar data given by Aldis [1] to 21D, for the range $x = 0.1(0.1)6.0$. No discrepancies were detected.

The range, precision, and accuracy of the tables under review establish them as the definitive tables of the Kelvin functions at the present time.

J. W. W.

1. W. STEADMAN ALDIS, "On the numerical computation of the functions $G_0(x)$, $G_1(x)$, and $J_n(x\sqrt{i})$," Roy. Soc. London, *Proc.*, v. 66, 1900, p. 32–43.

10[L].—NUMERICAL COMPUTATION BUREAU, Report No. 11, *Tables of Whittaker Functions (Wave Functions in Coulomb Field) Part 2*, The Tsuneta Yano Memorial Society, 1–9 Yuraku-cho, Chiyoda-Ku, Tokyo, Japan, 1959, 52 p., 26 cm. Price $3.00.

The first part of these tables was reviewed in *MTAC*, v. 12, 1958, p. 86–88. The earlier review contains some errors and fails to give complete information.

The functions considered are defined thus:

$$G_{\xi,l} + iF_{\xi,l} = \exp\left(-\frac{x}{2}\xi\right)\exp\left[-i\left(\frac{l}{2}\pi - \sigma_l\right)\right] W_{i\xi,l+1/2}(-ix).$$

The earlier review has $\frac{1}{2}$ in place of $l + \frac{1}{2}$ in the subscript of the Whittaker function, W, and incorrectly uses $G_{\xi,1/2}(x)$ and $F_{\xi,1/2}(x)$ for the case $l = 0$. Both the previous review and the present tables fail to define $\sigma_l = \arg \Gamma(l + 1 + \frac{1}{2}ix)$.

The table now reviewed is concerned with values of $F_{\xi,0}(x)$ for large ξ. Values of this function are presented to five decimal places, together with δ_r^2 and δ_k^2, where $k = 1/\xi$ and $r = x/(2k)$, corresponding to $r = 0(.1)10$ and $k = 0(.01)1$, that is, $\xi \geqq 1$.

It is observed that for $k = 0$, that is, $\xi \rightarrow \infty$,

$$F_{\xi,0}(x) = \sqrt{2r}J_1(2\sqrt{2r}); \qquad G_{\xi,0}(x) = \sqrt{2r}Y_1(2\sqrt{2r}),$$

which are identifiable as Bessel-Clifford functions multiplied by $2r$. This relation allows comparison with appropriate data in a publication [1] of the National Bureau of Standards; such a comparison has revealed 24 errors (all due to rounding) in the tables under review, thereby suggesting a general accuracy therein within one unit of the fifth decimal.

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1. NBS Applied Mathematics Series, No. 28, *Tables of Bessel-Clifford Functions of Orders Zero and One*, U. S. Government Printing Office, Washington, D. C., 1953.

11[L, M].—W. W. Gerbes, G. E. Reynolds, M. R. Hoes, & C. J. Drane, Jr., *Table of S(x) and its First Eleven Derivatives*, Vol. 1, 2, 3, Air Force Cambridge Research Center, Bedford, Massachusetts, 1958, 27 cm.

The tabulated function $S(x)$ defined by

$$S(x) = \int_0^x \left(\frac{\sin \frac{u}{2}}{u/2} \right)^2 du$$

is related to the sine integral $Si(x)$ by

$$S(x) = 2\left[Si(x) - \frac{1 - \cos x}{x} \right].$$

For ease in computation in the design of antennas, the function $S(x)$ and its first eleven derivatives are tabulated to six decimal places for

$$x = 0°(1°)18{,}000°.$$

The introduction gives the characteristics of the functions, reduction formulas, power series representations, asymptotic expressions, integral representations, differential equations, transforms, addition formulas, etc., and the method of computation.

The tables were computed using the IBM 650 calculator.

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